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Problem	Problem Name	Time Limit	Memory Limit
A	f(Graph)	5 s	256 MB
B	Substrings	1 s	256 MB
C	Coloring Points	3 s	256 MB
D	Integer Division	3 s	256 MB
E	Counting Subsequence	1 s	256 MB
F	Operations on Tree	3 s	256 MB
G	Birthday Problem	5 s	256 MB
H	Diameter	3 s	256 MB
I	Counting Substring	1 s	256 MB
J	Almost Convex Hull	2 s	256 MB

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A. $f(\text{Graph})$

For an edge-weighted undirected graph $G(V, E)$, where V is the set of vertices of G and E is the set of edges of G, $f(G)$ is defined as:

- $f(G)$ has $|E|$ vertices where i-th vertex corresponding to the i-th edge in G.
- There is a weighted edge between x and $y (x \neq y)$ in $f(G)$ if and only if x-th edge and y-th edge in G have at least one common vertex and its weight is equal to the sum of the weight of x-th edge and y-th edge.

Let $d(G, i, j)$ be the length of shortest path between i-th vertex and j-th vertex in G. The length of a path is the sum of the weight of each edge.

Given an edge-weighted undirected **complete** graph $G(V, E)$.

Please calculate $\sum_{i=1}^{|E|} \sum_{j=i+1}^{|E|} d(f(G), i, j)$

Input

The first line of input contains an integer N indicating the size of V in G. Following N lines each contains N space-separated integers w_{ij} indicating the weight of the edge between i and j. **Note that the value of w_{ii} is meaningless.**

- $1 \leq N \leq 500$
- $0 \leq w_{ij} \leq 10^9$
- $w_{ij} = w_{ji}$
- $w_{ii} = 0$

Output

Output one line containing a single number indicating the required value. Since the number may be to large, please output it module 998244353.

Sample Input 1	Sample Output 1
2 0 1 1 0	0

Sample Input 2	Sample Output 2
3 0 1 1 1 0 1 1 1 0	6

Sample Input 3	Sample Output 3
4 0 1 1 1 1 0 2 2 1 2 0 3 1 2 3 0	56

B. Substrings

Given a $N \times M$ matrix A, the element on the i-th row, j-th column is in color c_{ij} and has weight w_{ij}.

You can choose a $P \times Q$ sub-matrix B from A with cost
$(P+1) \times (Q+1) \times$ (maximum weight in the corresponding sub-matrix).

Then, an infinite matrix C will be generated from B where the element on the i-th row, j-th column in C will be equal to the element on ($i \mod P$)-th row, ($j \mod Q$)-th column in B.

Find the minimum cost to choose B such that matrix A is a sub-matrix of matrix C when considering only color.

Input

The first line of input contains two integers N, M indicating the size of matrix A. Following N lines each contains M lowercase English letters where the j-th character on the i-th line is c_{ij}. Following N lines each contains M space-separated integers where the j-th integer on the i-th line is w_{ij}.

- $1 \leq N \times M \leq 10^6$
- c_{ij} is an lowercase English letter(`"a-z"`)
- $0 \leq w_{ij} \leq 10^9$

Output

Output one line containing a single number indicating the minimum cost.

Sample Input 1	Sample Output 1
2 5 acaca acaca 3 9 2 8 7 4 5 7 3 1	18

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C. Coloring Points

There are N points with integer coordinates in 2D system, which form a set P. Given a positive integer K, define $f(P, K)$ be the number of way to color those points with K different color. For a coloring, each point must be colored into exactly one of those K colors, some points may be in the same color, and some colors may not be used.

Two colorings A, B are considered the same if and only if exists a point o (could have real value coordinates) and angle α such that after rotating those points in A α degree counterclockwise by the center o, the resulting positions and colors are exactly the same as B.

Directly calculating $f(P, K)$ may be too hard, you just need to compute $\sum_{P' \subseteq P, |P'|>0} f(P', K)$.

Input

The first line of input contains two space-separated integers N, K indicating the number of points and given integer K. Following N lines each contains two space-separated integers x_i, y_i indicating the coordinates of i-th points.

- $1 \leq N \leq 3000$
- $1 \leq K \leq 10^9$
- $|x_i|, |y_i| \leq 10^8$
- $(x_i, y_i) \neq (x_j, y_j)$ if $i \neq j$

Output

Output one line containing a single number indicating the required value. Since the number may be to large, please output it module 998244353.

Sample Input 1	Sample Output 1
2 2 1 0 0 1	7

Sample Input 2	Sample Output 2
5 3 0 0 2 0 0 2 2 2 1 1	747

D. Integer Division

Given N integers, you need to divide them into M groups. The i-th group must contain at least l_i integers and at most r_i integers. And, each integer must be assigned to exactly one of the group.

The value of a group is the average of its elements. Find the maximum possible total value of M groups composed of given N integers.

Input

The first line of input contains an integer N indicating the number of integers. The second line of input contains N space-separated integer a_i indicating the given integers. The third line of input contains an integer M indicating the required number of groups. Following M lines each contains two space-separated integer l_i, r_i indicating the size of i-th group must in $[l_i, r_i]$.

- $1 \leq N \leq 250$
- $1 \leq M \leq 30$
- $1 \leq a_i \leq 10^6$
- $0 \leq l_i \leq r_i \leq N$
- It's guaranteed that there exists at least one valid assignment.

Output

Output one line containing a single number indicating the required value. You answer will be considered correct if its absolute or relative error is within 10^{-6}.

Sample Input 1	Sample Output 1
5 3 6 1 2 5 3 1 3 1 1 2 3	13.000000000000000000

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E. Counting Subsequence

Given a sequence A with length N where $1 \leq A_i \leq K$. Count number of distinct subsequences of A with length $N - M$.

Input

The input consists of several test cases and is terminated by EOF.

The first line of input contains three space-separated integer N, M, K. The second line of input contains N space-separated integer A_i.

- $1 \leq N \leq 10^5$
- $1 \leq M \leq \min(n - 1, 10)$
- $1 \leq K \leq 10$
- $1 \leq A_i \leq K$
- Sum of N does not exceed 10^6

Output

For each test case, output one line containing a single number indicating the required value. Since the number may be to large, please output it module $1000000007(10^9 + 7)$.

Sample Input 1	Sample Output 1
3 2 2 1 2 1 4 2 2 1 2 1 2	2 4

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F. Operations on Tree

Given a vertex-weighted rooted tree with N vertices where root is the vertex numbered 1.

Define the value of a connected component is the product of the weights of its vertices.

Please support following operations:

- `0 x y`: change the weight of vertex x into y
- `1 x y`: change the father of vertex x into y
- `2 x y`: find the sum of value of all connected components containing vertex x with size y.

Input

The first line of input contains two space-separated integers N, M indicating the size of the rooted tree and number of operations. The second line of input contains N space-separated integers w_i indicating the weight of i-th vertex. The third line of input contains $N-1$ space-separated integers p_i indicating the father of 2-nd vertex to N-th vertex. Following M lines each contains three space-separated integers c, x, y indicating the corresponding operation.

- $1 \leq N, M \leq 10^5$
- $1 \leq w_i < 1000000007(10^9+7)$
- $0 \leq c \leq 2$
- If $c = 0$, $1 \leq x \leq N, 1 \leq y < 1000000007(10^9+7)$
- If $c = 1$, $1 \leq x, y \leq N$ and y is not in the subtree of x
- If $c = 2$, $1 \leq x \leq N, 1 \leq y \leq 10$

Output

For each query operation($c = 2$), output one line containing a single number indicating the required value. Since the number may be to large, please output it module $1000000007(10^9+7)$.

Sample Input 1	Sample Output 1
3 3 1 2 3 1 1 2 1 2 2 1 3 2 2 3	5 6 6

G. Birthday Problem

There are N months in a year and M days in each month.

Given a non-empty set of days S containing Eddy's birthday, Alice, Bob, and Chris only know this information. Let say Eddy's birthday is on y-th day of x-th month.

Eddy tells Alice the value of x and tells Bob the value of y. After that, following scenario happens exactly K times: Alice says that she doesn't know Eddy's birthday after hearing the last statement(if any) and Bob says that he doesn't know Eddy's birthday after Alice's last statement.

After that scenario happened K times, Alice said that she knows Eddy's birthday. After hearing that, Bob said that he knows Eddy's birthday as well (Note that he might have figured it out before Alice's statement). Finally, after hearing both statement, Chris says that he knows Eddy's birthday(possibly before some statements are said).

Assuming all of them(possibly except Eddy) are perfect logicians and do not lie, among the $2^{N\times M}-1$ possible non-empty sets of days S, how many of them are there such that the scenario stated above is possible.

Input

The first line of input contains three space-separated integer N, M, K.

- $1 \leq N, M, K \leq 30$

Output

Output one line containing a single number indicating the required value. Since the number may be to large, please output it module $1000000007(10^9+7)$.

Sample Input 1	Sample Output 1
3 3 1	18

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H. Diameter

Given a tree with N vertices, find the number of vertices subsets S where diameter of S is exactly D.

The distance of two vertices x, y is the minimum number of edges we need to go through from x to y. The diameter of a subset S of vertices is the maximum distance between two elements in S. If the size of S is less than 2, the diameter is defined as 0.

Input

The first line of input contains an integer N indicating the number of vertices. Following $N-1$ lines each contains two space-separated integers u_i, v_i indicating there is an edge between vertex u_i and vertex v_i. The last line of input contains an integer D indicating the required diameter.

- $1 \leq N \leq 10^5$
- $1 \leq u_i \neq v_i \leq N$
- $1 \leq D \leq N$
- It's guaranteed that given input forms a tree.

Output

Output one line containing a single number indicating the required value. Since the number may be to large, please output it module $1000000007(10^9 + 7)$.

Sample Input 1	Sample Output 1
4 2 3 1 3 3 4 2	8

Sample Input 2	Sample Output 2
4 2 3 1 3 3 4 1	3

I. Counting Substring

Given a string S with length N, find out the maximum possible subset of its substrings($\frac{N(N+1)}{2}$ such many substrings) such that no two of them are similar.

Two string A, B are similar if and only if $|A| = |B|$ and there exists a function f such that $A_i = f(B_i) \forall i \in [0, |A| - 1]$, where f must satisfy $f(x) \neq f(y)\ \forall x \neq y$.

Input

The input consists of several test cases and is terminated by EOF. The first line of input contains an integer N indicating the length of S. The second line of input contains a string S with length N.

- $1 \leq N \leq 5 \times 10^4$
- $s_i \in$ `"abc"`
- Sum of N does not exceed 2×10^5

Output

For each test case, output one line containing a single number indicating the size of maximum possible subset satisfying the requirement.

Sample Input 1	Sample Output 1
4 abaa 4 abab	6 4

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J. Almost Convex Hull

Given N points in 2D coordinates system, find the number of distinct almost convex hull whose vertices all belong to the given points.

A polygon is called convex if every line segment that connects two non-adjacent vertices is a diagonal.

A diagonal is a line segment that connects two non-adjacent vertices and lies completely inside the polygon

A polygon is called almost convex hull if it is convex or there exists a diagonal that divides it into two convex hulls.

Input

The first line of input contains an integer N indicating the number of points. Following N lines each contains two space-separated integers x_i, y_i indicating the coordinates of i-th point.

- $3 \leq N \leq 15$
- $|x_i|, |y_i| \leq 100$
- No three given points will lie on the same straight line

Output

Output one line containing a single number indicating the required value.

Sample Input 1	Sample Output 1
4 -2 -2 2 -2 -2 2 2 2	5

Sample Input 2

```
5
-2 -2
2 -2
-2 2
2 2
3 0
```

Sample Output 2

```
16
```

Sample Input 3

```
5
-2 -2
2 -2
-2 2
2 2
0 1
```

Sample Output 3

```
22
```