2021 NTU ICPC Team Preliminary

National Taiwan University

2021/08/21

Language	Version	Compile Flags	Extensions
C++	g++ 9.4.0	`-O2 -std=c++17 -static`	`.cc, .cpp`

Problem	Problem Name	Time Limit	Memory Limit
A	Almost-Equal Substrings	2 s	256 MB
B	Broken Testdata	15 s	256 MB
C	Constrained Optimization	2 s	256 MB
D	Divisible Tuples	4 s	256 MB
E	Epic Permutations	2 s	256 MB
F	Find the Sum	3 s	256 MB
G	Good Subsets	1 s	256 MB
H	How Dare You	3 s	256 MB
I	Independent Sets	2 s	256 MB
J	Just Polygons	6 s	256 MB
K	Kitty Town and Meow Coin	1 s	256 MB

This page is intentionally left blank.

A. Almost-Equal Substrings

Two strings x and y are considered *almost equal* if $|x| = |y|$ and the Hamming distance between x and y is at most one. That is, there exists at most one $1 \leq i \leq |x|$ such that $x_i \neq y_i$.

Given two strings s and t, please find the longest substring s' of s and t' of t such that s' and t' are almost equal.

Input

The first line of the input contains the string s while the second line contains t. Both s and t consist of lowercase English letters.

- $1 \leq |s|, |t| \leq 10^5$

Output

Output two lines. The first line should be s' and the second line should be t'. If there are multiple solutions, you can output any one of them.

Sample Input 1	Sample Output 1
abcdef pwbzdeq	bcde bzde

Sample Input 2	Sample Output 2
abcde bcd	bcd bcd

Sample Input 3	Sample Output 3
ntuicpcpreliminary almostequalsubstrings	min rin

This page is intentionally left blank.

B. Broken Testdata

You are preparing the following easy problem for the upcoming contest: Given an array a of length N, please calculate the greatest common divisors (GCD) for all pairs of adjacent elements in a (including the one consisting of a_1 and a_N)

Unfortunately, the judging machine crashed and all the testdata you generated were lost. Luckily, you still had a copy of the output file in the backup system. Can you recover the original input file from it, or report that the output file is corrupted as well?

Formally speaking, given an array b of length N, find an array a of the same length such that $b_i = \gcd(a_i, a_{i+1})$ for $1 \leq i < N$ and $b_N = \gcd(a_N, a_1)$, or state that such array is non-existent.

Input

The first line of the input contains an integer T, which is the number of test cases.

For each test case, there are two lines. The first line contains an integer N, which is the length of the arrays a and b. The second line contains N space-seperated integers, which represent $b_1, b_2, \ldots, b_N$, respectively.

- $T \leq 100000$
- The sum of N over all test cases does not exceed 21097560
- $1 \leq N \leq 10^5$
- $1 \leq b_i \leq 10^9$ for all valid i's

Output

For each test case, if the output file is corrupted, print `-1` in a single line.

Otherwise, print n space-seperated integers $a_1, a_2, \ldots, a_n$ in a single line. Your solution should additionally satisfy the constraint that $a_i \leq 10^{18}$ for all valid i's. It can be proved that the constraint can be achieved if there is at least one solution. If there are multiple solutions, you can output any one of them.

Sample Input 1	Sample Output 1
1 3 2 6 4	4 18 24

C. Constrained Optimization

Given an array $a_1, a_2, \ldots, a_n$ of n non-negative integers and a permutation $p_1, p_2, \ldots, p_n$, please find an array $b_1, b_2, \ldots, b_n$ of *non-negative* real numbers such that,

- $b_i + b_{p_i} \leq a_i$ for all $1 \leq i \leq n$, and
- the sum $b_1 + b_2 + \cdots + b_n$ is maximized.

Input

The first line of the input contains an integer n, representing the length of the arrays. The second line contains n integers $p_1, p_2, \ldots, p_n$ while the third lines contains n integers $a_1, a_2, \ldots, a_n$.

- $1 \leq N \leq 10^6$
- $0 \leq a_i \leq 10^9$
- It's guaranteed that p is a permutation

Output

Output the maximum possible sum of array b as a real number. Your answer will be considered correct if the absolute or relative error between your answer and judge's answer is less than 10^{-6}.

Notes

For the first sample case, the optimal solution is $b_1 = b_2 = b_3 = 0.5$.

Sample Input 1	Sample Output 1
3 2 3 1 1 1 1	1.500000000

Sample Input 2

```
10
10 1 3 6 8 5 7 4 2 9
2 4 10 2 3 1 4 2 4 4
```

Sample Output 2

```
16.000000000
```

D. Divisible Tuples

Given a prime number p and a positive integer k, please calculate the number of k-tuples of integers $(x_1, ..., x_k)$ such that

- $\sum_{i=1}^{k} x_i^2$ is divisible by p
- $0 \le x_i < p$ for all i's

Please output the number of such k-tuples modulo $10^9 + 7$.

Input

The first line of the input contains an integer T, which is the number of test cases.

For each test case, there is a line containing two positive integers p and k.

- $T \le 1000000$
- p is a prime
- $1 \le p, k \le 10^9$

Output

For each test case, output a line containing the answer to the test case.

Sample Input 1	Sample Output 1
2 3 1 5 3	1 25

This page is intentionally left blank.

E. Epic Permutations

Let p be a permutation of length N. We say that p is epic if $p_i + p_{i+1} \leq N + 1$, for all $1 \leq i < N$.

Please find the number of epic permutations of length N, modulo 998244535.

Input

The first line of the input contains a single integer T, the number of test cases.

For each test case, there is one line, which contains a positive integer N.

- $T \leq 40$
- $1 \leq N \leq 40$

Output

For each test case, output a single line containing the answer to the test case.

Notes

For $N = 1$, the only permutation of length N is $\{1\}$ and it is epic.

For $N = 2$, both of the permutations $\{1, 2\}$ and $\{2, 1\}$ are epic.

For $N = 3$, only permutations $\{2, 1, 3\}$ and $\{3, 1, 2\}$ are epic.

Sample Input 1	Sample Output 1
3 1 2 3	1 2 2

This page is intentionally left blank.

F. Find the Sum

f is a function satisfying the following conditions:

- $f(0) = 0$
- $f(n) = \min\{f(n - i^2) + 1 \mid n \geq i^2, i \in \mathbb{N}\}$

For example, $f(1) = 1, f(2) = 2, f(3) = 3, f(4) = 1, f(5) = 2$.

Given two positive integers L, R with $L \leq R$, please calculate

$$\sum_{i=L}^{R} f(i)$$

.

Input

The only line of the input contains two positive integers L and R.

- $1 \leq L \leq R \leq 10^9$

Output

Output the answer to the problem in a single line.

Sample Input 1	Sample Output 1
6 6	3

Sample Input 2	Sample Output 2
7 7	4

Sample Input 3	Sample Output 3
2 10	20

This page is intentionally left blank.

G. Good Subsets

Given a nonnegative integer m, a set of positive integers is considered to be $m-$good if the xor (exclusive-or) sum of the integers in the set is m. The quality of such set is defined as the sum of its elements.

Given a positive integer n, find the sum of qualities among all $m-$good subsets of the set $\{1, 2, ..., n\}$.

Please output the number modulo $10^9 + 7$.

Input

The first line of the input contains a single integer T, the number of test cases.

T lines follow, and each of which contains two integers n and m, describing a test case.

- $1 \leq T \leq 10^5$
- $1 \leq n \leq 10^9$
- $0 \leq m \leq 10^9$

Output

Foe each of the T test cases, output a single line containing the number of m-good subsets modulo $10^9 + 7$.

Notes

For $n = 1$ and $m = 1$, the only m-good subset of $\{1\}$ is $\{1\}$. The quality of that subset is 1.

For $n = 3$ and $m = 2$, there are two m-good subsets of $\{1, 2, 3\}$: $\{1, 3\}$ and $\{2\}$. The former has quality 4 and the latter has quality 2. The sum of qualities is 6.

Sample Input 1	Sample Output 1
2 1 1 3 2	1 6

H. How Dare You

A cactus is a connected undirected graph in which every edge belongs to at most one simple cycle. You are given a cactus G consisting of N vertices and M edges, where vertices are numbered from 1 to N. You have to answer Q queries of the following form:

- Given s and t, what is the length of the shortest path between vertex s and vertex t? Also, what is the number of such shortest paths?

The length of a path is the number of edges it comprises. Two paths are considered different if there exists an edge in G belonging to one path but not the other.

Input

The first line of the input contains two integers N and M — the number of vertices and the number of edges. M lines follow, and the i-th of which contains two integers u_i and v_i, denoting the endpoints of the i-th edge.

The next line contains a single integer Q, the number of queries. Finally, Q lines follow. Each of the Q lines contains two integers s and t, representing a query.

- $1 \leq N \leq 10^5$
- $1 \leq u_i, v_i \leq N$
- $u_i \neq v_i$
- $1 \leq s, t \leq N$
- $1 \leq Q \leq 10^5$

Output

For each of the query, output two integers in a single line: the shortest distance between s and t, followed by the number of such shortest paths modulo $10^9 + 7$.

Sample Input 1

```
3 3
1 2
2 3
3 1
3
1 2
2 3
3 1
```

Sample Output 1

```
1 1
1 1
1 1
```

Sample Input 2

```
5 5
1 2
2 3
3 4
4 1
3 5
1
1 5
```

Sample Output 2

```
3 2
```

I. Independent Sets

You are given an undirected simple (but not necessarily connected) graph G of N vertices. The vertices in G are numbered from 1 to N. Please find an independent set S such that

- $|S| \geq \sum_{v \in V(G)} \frac{1}{\deg(v)+1}$

The set of all vertices in G is denoted as $V(G)$.

A vertex set $S \subseteq V(G)$ is said to be independent if no two vertices in S are adjacent. The degree $\deg(v)$ of vertex v is the number of edges incident to v.

Input

The first line of the input contains two integers N and M — the number of vertices and the number of edges. M lines follow, and the i-th of which contains two integers u_i and v_i, denoting the endpoints of the i-th edge.

- $1 \leq N \leq 2 \times 10^5$
- $0 \leq M \leq \min(\frac{N \times (N-1)}{2}, 3 \times 10^5)$
- $1 \leq u_i, v_i \leq N$
- $u_i \neq v_i$

Output

If no such independent set exists, output `NO`.

Otherwise, output `YES` followed by the description of the independent set in the next two lines.

The first line of the description should contain an integer $|S|$, the size of the independent set. The second line of the description should contain $|S|$ space-separated distinct integers between 1 and N, representing the vertices in S. If there are multiple solutions, you can output any one of them.

Notes

In the sample case, the vertex set $\{1, 3, 5\}$ is an independent set of size at least

$$\sum_{v \in V(G)} \frac{1}{\deg(v)+1} = \frac{1}{2} + \frac{1}{4} + \frac{1}{3} + \frac{1}{4} + \frac{1}{3} + \frac{1}{2} = \frac{13}{6}$$

Sample Input 1	Sample Output 1
6 6 1 2 2 3 3 4 4 5 2 5 4 6	YES 3 1 3 5

J. Just Polygons

You are given two convex polygons on the 2D plane. Please find the area of their union.

Input

The first line of the input contains an integer T, which is the number of test cases.

For each test case, the first line contains an integer N, which indicates the number of points in the first polygon. Each of the next N lines contains two integers, the x-coordinate and the y-coordinate of a point in the first polygon.

The next line contains an integer M, which indicates the number of points in the second polygon. Each of the next M lines contains two integers, the x-coordinate and the y-coordinate of a point in the second polygon.

The points in a polygon are given in counter-clockwise order.

- $T \leq 1000$
- $1 \leq N, M \leq 20001$
- The sum of N and M over all test cases does not exceed 2026344
- The absolute values of all coordinates do not exceed 10^8

Output

For each test case, output the area of the union in a single line. Your answer will be considered correct if the absolute or relative error between your answer and judge's answer is less than 10^{-6}.

Sample Input 1	Sample Output 1
1 4 0 0 2 0 2 2 0 2 3 3 3 0 3 3 0	8.00000000880301

K. Kitty Town and Meow Coin

Kitty Town is formed by a $N \times M$ grid, and each cell has a non-negative integer value $A_{i,j}$, the number of Meow Coins at coordinate (i, j).

Now, you want to collect all the Meow Coins in Kitty Town by performing the following operations many times.

For each operation, you have to walk from $(1, 1)$ to (N, M), and there are only two directions you can go: right and down. That is to say, if you are at coordinate (i, j), then the next coordinate you can go to is either $(i + 1, j)$ or $(i, j + 1)$. Additionally, you can choose whether to pick up exactly one Meow Coin or not for each of the coordinate in the current path from $(1, 1)$ to (N, M).

Now, can you tell me what is the minimum number of operations you need to do in order to collect all the Meow Coins?

Input

The first line of the input contains two integers N and M, representing the number of rows and the number of columns in Meow Kitty.

For the next N lines, each line contains M integers $A_{i,j}$, representing the number of Meow Coin in coordinate (i, j).

- $1 \leq N \times M \leq 10^6$
- $0 \leq A_{i,j} \leq 10^9$

Output

Output one integer in a line, which is the minimum number of operations you need to perform in order to collect all the Meow Coins.

Sample Input 1	Sample Output 1
2 2 0 1 2 1	3

This page is intentionally left blank.